Table 2 Comparison of methods with $n = 4$ satellites, $h_P = 150$ naut miles, and $h_T = 8000$ naut miles

	ΔV requirements, fps			
Method	Can be supplied by booster	Must be supplied by satellite	Total	Time to establish system, hr
PE, TA	8710	1,610	10,320	21.44
PE, PP	2530	7,790	10,320	8.66
HP	0	10,320	10,320	3.64
FP[a]	0	11,000	11,000	3.28

[a] The values given correspond to an arbitrarily chosen point from Table 1.

period t_W between satellite expulsions is not a function of the transfer orbit method. Therefore, Eq. (7) is valid for the present method as well as the HP method. For the example carried throughout the analysis, a comparison between the four methods just described is presented in Table 2.

Note the variance in time required to establish the system for approximately equal values of ΔV_{total}. The tradeoff is between time required to establish the system and the fraction of the velocity requirements that can be supplied by the boost vehicle.

Reference

[1] Lüders, R. D., "Satellite networks for continuous zonal coverage," ARS J. **31**, 179–184 (1961).

Large Thermal Deflections of Thin-Walled Metal Tubes in Space

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Nomenclature

- E = Young's modulus
- h = cylinder wall thickness
- I = moment of inertia
- k = thermal conductivity
- l = length of tube
- M_0 = moment independent of γ
- r = radius of tube
- R = radius of curvature of tube
- s = arc length
- S = solar constant
- T = absolute temperature
- $\bar{T}$ = mean radiant temperature, $(S\bar{\alpha}/\pi\sigma\bar{\epsilon})^{1/4}$
- T_0 = minimum temperature of cross section
- α_1 = circumferential conductance parameter, $kh/(r^2\sigma\bar{\epsilon}\bar{T}^3)$
- α_2 = longitudinal conductance parameter, $kh/(R^2\sigma\bar{\epsilon}\bar{T}^3)$
- $\bar{\alpha}$ = over-all solar absorptivity
- β = radiation parameter $\epsilon_i/\bar{\epsilon}$,
- γ = angle between local normal and sun vector
- ϵ = over-all infrared emissivity
- θ = cylindrical coordinate, $0 < \theta < \pi$
- ν = coefficient of thermal expansion
- σ = Stefan-Boltzmann constant
- τ = dimensionless temperature
- τ_0 = lowest value of τ at a given cross section

Introduction

LONG, thin-walled metal tubes are used on satellites for certain functions, e.g., as booms for gravity-gradient stabilization. Uneven solar heating can cause large thermal deflections in these tubes.

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In this note, a procedure is outlined for calculating the deflection for a symmetrical but otherwise arbitrary temperature distribution through each particular cross section. The assumptions are as follows: temperature gradient through the thickness of the tube wall is negligible, internal radiation is of lesser importance than circumferential conduction, for each cross section there is a mean radiant temperature proportional to $\cos^{1/4}\gamma$, and the effects of longitudinal conduction are negligible in comparison with circumferential conduction. The first two assumptions can be justified readily for beryllium tubes of 1-in. diam and 0.005-in. wall thickness by a direct comparison of the parameters α_1 and β. The third assumption is introduced because, when the effects of circumferential conduction predominate, the local temperature should be determined from the relation implied in the Stefan-Boltzmann law. The final assumption is discussed in this paper.

Statement of Problem

The temperature equation can be stated in the dimensionless form as

$$-\alpha_1\frac{\partial^2\tau}{\partial\theta^2} - \alpha_2\frac{\partial^2\tau}{\partial\gamma^2} = \cos\gamma f(\theta) - (1+\beta)\tau^4 + \frac{\beta}{4}(g\gamma)\int_\theta^{\theta+2\pi}\tau^4(\theta')\sin\frac{\theta'-\theta}{2}\,d\theta'$$

where the derivatives indicate the effects of conduction in the circumferential and longitudinal directions, and the terms on the right-hand side are the direct solar-heat input, heat loss by its own radiation, and heat gain by internal radiation. If $0(\partial^2\tau/\partial\theta^2) = 0(\partial^2\tau/\partial\gamma^2)$, and $r \ll R$, then the second term on the left-hand side can be neglected in comparison with the first. Moreover, if $\alpha_1 > \beta$, then $g(\gamma) \approx 1$ can be set, and if $\alpha_1 \gg 1$, the problem reduces to the solution of the linear integrodifferential equation for a tube inclined at an angle of $90° + \gamma$ with respect to sun rays.[1] This solution depends upon the Fourier series representation of the solar-heat input function $S\cos^+\theta\cos\gamma$ with $\cos^+\theta$ = half-wave rectified cosine function:

$$\cos^+\theta = \frac{1}{\pi} + \frac{1}{2}\cos\theta + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n\cos 2n\theta}{1-4n^2}$$

To show the effect of a variable γ, one can here "delinearize" the solution from Ref. 1 by writing $4\tau - 3 \approx \tau^4$; then, after a few transformations, one has finally $\Delta\tau(\theta,\gamma) = \Delta\tau(\theta)\cos^{1/4}\gamma$. The solution may be rewritten as

$$(\tau-\tau_0)(\cos\gamma)^{-1/4} = \left(\frac{\pi}{8}a_0 + \frac{\pi}{4}\sum_{n=1}^{\infty}a_n\cos n\theta\left\{\frac{1}{4}\alpha_1 n^2 + [1-(1+\beta)4n^2](1-4n^2)^{-1}\right\}^{-1}\right)\int_\pi^\theta \tag{1}$$

As long as $R \gg r \gg h$ holds, the simple theory of strength of materials may be expected to give correct results, so that the equation of the elastic line (Fig. 1) will be $d\gamma/ds = M/EI$. Here the moment M is caused entirely by the thermal loading. In view of Eq. (1), let $M_0 = M/(EI\cos^{1/4}\gamma)$, or

$$M_0 = \frac{\nu}{I}\int_A (T-T_0)y\,dA \tag{2}$$

Equation (2) neglects the direct effect of solar pressure†; since

$$d\gamma/ds = M_0\cos^{1/4}\gamma \qquad \text{for } \gamma_0 = 0 \qquad \text{and } s(0) = 0$$

$$\int_0^\gamma \sec^{1/4}\eta\,d\eta = M_0 s \tag{3}$$

† For small deflections, the effect of solar pressure can be calculated by the expression $y = prl^4/4EI$.

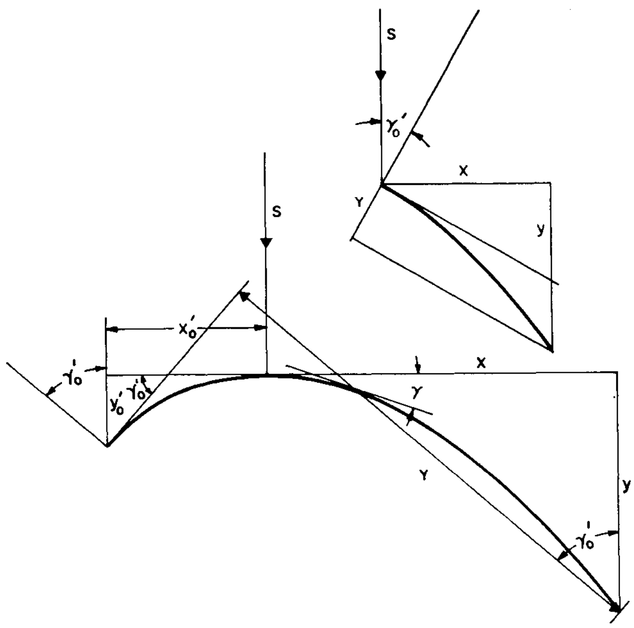

Fig. 1 Generalized geometry of large deflection, various cases.

which relates the slope and the length of the arc s. From Fig. 1, $dy/ds = \sin\gamma$, $dx/ds = \cos\gamma$, and by the latter, for $\gamma_0 = 0$ at $x = 0$,

$$\int_0^{\gamma} \cos^{3/4}\eta \, d\eta = M_0 x \tag{4}$$

which relates the slope to the distance from the origin. Similarly, the slope and the deflection are related by

$$y - y_0 = 4(\cos^{3/4}\gamma_0 - \cos^{3/4}\gamma)/3M_0 \tag{5}$$

In the present case, when $\gamma_0 = 0$, then $y_0 = 0$ also applies. To determine $y(x)$, one finds first the value of γ that satisfies Eq. (4). Then, using Eq. (5), one obtains the desired deflection. The integrals indicated in Eqs. (3) and (4) are represented graphically in Fig. 2; as a check on their numerical integration, it may be observed that, for $n > -1$,

$$\int_0^{\pi/2} \cos^n\eta \, d\eta = \frac{1}{2}(\pi)^{1/2}\Gamma\left(\frac{n+1}{2}\right)\Big/\Gamma\left(\frac{n}{2}\right) + 1$$

The foregoing relations also may be applied for cases where $\gamma_0 \neq 0$. First, let $\gamma_0 \neq 0$ be γ_0'. From Fig. 1, it is seen that then the true deflection is

$$Y = (y - y_0')\cos\gamma_0' + (x + x_0')\sin\gamma_0' \tag{6}$$

if condition $\gamma = 0$ applies for a location on the tube. If this condition is not satisfied although $\gamma_0' \neq 0$, then γ_0' has to be used as the lower limit in Eqs. (3) and (4). In this case

$$Y = y\cos\gamma_0' - x\sin\gamma_0' \tag{7}$$

In Eqs. (6) and (7), Y is the true deflection perpendicular to the tangent to the tube at its true orign.

Discussion of Results

The formula for a linear temperature distribution over the cross section, with a direct dependence of energy input on $\cos\gamma$ and for the normal incidence of sun rays at origin, is[2]

$$-y = M_0^{-1}\ln(\cos M_0 x) \tag{8}$$

As will be shown, the preceding equation reduces to the result where $\cos^{1/4}\gamma$ is used for small deflections ($\gamma \ll 1$). Then, $ds \approx dx$, $y \approx M_0 x$, and it follows from Eq. (5) that $y \approx \frac{1}{2}M_0x^2$. When, in addition, the temperature varies linearly across the diameter of the tube, then

$$M_0 = \frac{(T_{\max} - T_0)}{2Ir}\int_A y^2 dA = \frac{\nu}{2r}\Delta T_{\max} \tag{9}$$

and the expression for small deflection becomes

$$y = \nu\Delta T_{\max}x^2/4r \tag{10}$$

By plotting the linearized solution of the problem for $\alpha_1 > 10$, it appears[1] that T is very nearly a linear function of θ: $(T - T_0)/\Delta T_{\max} = 1 - \theta/\pi$, and

$$M_0 = 8\nu\Delta T_{\max}/2\pi^2 r \tag{11}$$

It is interesting to note that, in their experiments on 6-ft-long tubes, Kemper and Farrel[2] found that the deflection was 0.8 of the value calculated for T linear with y which agrees with Eq. (11). Since, by symmetry, an actual temperature curve must have a horizontal tangent at $\theta = 0$ and $\theta = \pi$, temperature distribution linear with θ gives indeed only a lower estimate of M_0

The maximum temperature differential may be obtained from Eq. (1) as

$$\Delta T_{\max} = \bar{\alpha}S\left\{\frac{kh}{r^2} + \frac{4\bar{\alpha}S}{\pi T}\left[\frac{4}{3}(1+\beta) - \frac{1}{3}\right]\right\}^{-1} \tag{12}$$

Then, if kh/r^2 is large in comparison with the other term in the denominator of Eq. (12), Eq. (10) becomes identical with the expression given in Ref. 2 for small deflections.

Numerical Example

Consider now a numerical example of thermal bending of a tube with $h = 5 \times 10^{-3}$ in., $k = 65$ Btu/(hr-ft-°F), $r = 0.5$ in., $\bar{\alpha} = 0.67$, and $\bar{\epsilon} = 0.33$, if $S = 443$ Btu/(hr-ft²). Here, from Eq. (12), if $\beta = 0$, $\Delta T_{\max} = 10$°F. Also, from Ref. 3, $\nu = 9.4 \times 10^{-6}$/°F, and, by Eq. (9), $M_0 = 1.18 \times 10^{-3}$ ft^{-1}. Then, for $l = 100$ ft, use of $y \approx \frac{1}{2}M_0x^2$ gives $y = 5.90$ ft; for $y = f(\cos^{1/4}\gamma)$, Eq. (5) gives $y = 5.8$ ft, and if $y = f(\cos\gamma)$, Eq. (8) results in $y = 5.86$ ft. For $l = 1000$ ft, the results are 590, 513, and 491 ft, respectively. A tenfold reduction in $\Delta T_{\max}$ could be obtained by the use of suitable coatings,[1] in which case deflections would be reduced also, roughly by a factor of 10. Using for solar pres-

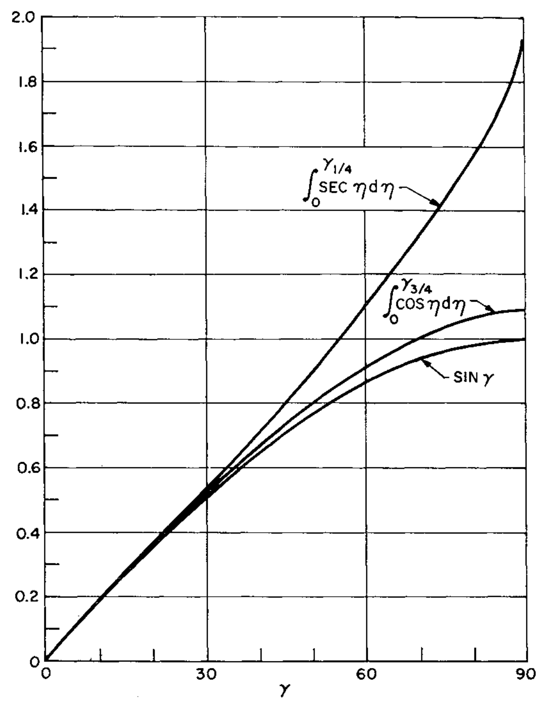

Fig. 2 Plot of Eqs. (3) and (4).

sure $p = 10 \times 10^{-5}$ dyne/cm^2, the resulting deflection is negligible in the cases already treated; for very slender rods, solar pressure effect may predominate because of dependence on I^{-1}.

Concluding Remarks

The discussion in this note allows one to draw conclusions on the plausibility of assumptions made by earlier investigators. The numerical results show that thermal deflections are relatively insensitive to the form of variation of the heat input function in the axial direction.

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Torsional Dynamics of an Axially Symmetric, Two-Body, Flexibly Connected Rotating Space Station

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Nomenclature

A, C = total moments of inertia of parts 1 and 2 about any axis in x, y plane, and about z axis, respectively
C_1, C_2 = moments of inertia of parts 1 and 2 about z axis
E = constant of integration (amplitude of γ)
f = viscous damping coefficient, torque/angular velocity
g = torque component applied to part 1 by part 2 along z direction
ϑ_1, ϑ_2 = inertia dyadics of parts 1 and 2 referred to x, y, z axes
$\mathbf{i}, \mathbf{j}, \mathbf{k}$ = unit vectors along the x, y, z body axes, respectively
$\mathbf{J}$ = applied angular impulse vector [see Eq. (19)]
K = torsional spring constant, torque/angular displacement
$\mathbf{L}$ = total system angular momentum vector
$\mathbf{N}$ = applied torque vector to system
$\mathbf{n}_{xy}$ = unit vector in direction of component of $\boldsymbol{\omega}_1$ on x, y plane
p = frequency of oscillation of γ
q = exponential decay constant
t = time
X, Y, Z = axes of the inertial coordinate system; Z axis coincides with angular momentum vector
x, y, z = principal axes of entire system, fixed in part 1 with origin at system center of mass; z axis is axis of revolution
γ = angular rotation between parts 1 and 2
ζ = constant of integration (phase angle associated with γ)
θ, φ, ψ = Euler angles in zZ, xy, and XY planes, respectively (Fig. 1); θ is precession half-cone angle
λ $\equiv \Omega_z(C - A)/A$
ω $\equiv \omega_x + i\omega_y$
$\boldsymbol{\omega}$ = angular velocity vector
Ω, Ω_z = constants of integration; Ω is complex

Subscripts and superscript

x, y, z = components along x, y, z axes; xy denotes component in xy plane
1, 2 = portion associated with or applied to part 1 or 2, respectively
0 = value immediately prior to applied impulse

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Introduction

As a result of the large size required for possible rotating manned space stations, as well as weight limitations on structural elements, these vehicles may be quite flexible. In this note the dynamic effects of concentric torsional oscillations of a vehicle (Fig. 1) are discussed. Constraints are considered to prevent relative motion of the two interconnected rigid bodies in any direction other than in concentric rotation about the axis of revolution. Thomson and Reiter[1] have shown that the general motion of a free elastic body of revolution results in a change in the precession cone angle. However, for the configuration considered herein, gyroscopic effects do not induce elastic vibration, and it is shown that the over-all precessional motion is independent of the relative oscillations of the two members.

Analysis

The center of mass of parts 1 and 2 (Fig. 1) need not coincide in the z direction. The axes are selected as fixed in part 1 or its imaginary extension (if the system center of mass does not fall within the confines of that part). The angular velocities are

$$\boldsymbol{\omega}_1 = \mathbf{i}\omega_x + \mathbf{j}\omega_y + \mathbf{k}\omega_z \qquad \boldsymbol{\omega}_2 = \boldsymbol{\omega}_1 + \mathbf{k}\dot{\gamma} \tag{1}$$

By summing the angular momentum of all of the particles and noting that the moments of inertia of part 2 are constants in the body reference frame, the system angular momentum is

$$\mathbf{L} = \vartheta_1 \cdot \boldsymbol{\omega}_1 + \vartheta_2 \cdot \boldsymbol{\omega}_2 = \mathbf{i}A\omega_x + \mathbf{j}A\omega_y + \mathbf{k}(C\omega_z + C_2\dot{\gamma}) \tag{2}$$

The components of the torque along the body reference axes are obtained by projecting the spatial derivative of $\mathbf{L}$ on this coordinate system:

$$\mathbf{N} = (d\mathbf{L}/dt) + \boldsymbol{\omega}_1 \times \mathbf{L} \tag{3}$$

For the case of the free system, $\mathbf{N}$ is zero. In addition, g is assumed to be a function of γ and $\dot{\gamma}$ only:

$$A\dot{\omega}_x + (C - A)\omega_y\omega_z + C_2\dot{\gamma}\omega_y = 0 \tag{4}$$

$$A\dot{\omega}_y - (C - A)\omega_x\omega_z - C_2\dot{\gamma}\omega_x = 0 \tag{5}$$

$$C\dot{\omega}_z + C_2\ddot{\gamma} = 0 \tag{6}$$

$$C_1\dot{\omega}_z = g(\gamma, \dot{\gamma}) \tag{7}$$

From Eqs. (6) and (7)

$$(C_1C_2/C)\ddot{\gamma} + g(\gamma, \dot{\gamma}) = 0 \tag{8}$$

$$\omega_z = \Omega_z - (\dot{\gamma}C_2/C) \tag{9}$$

where Ω_z is a constant of integration. With the use of this result, the solution to Eqs. (4) and (5) is

$$\omega/\Omega = \exp\{\textstyle\int i[\lambda + (\dot{\gamma}C_2/C)]dt\} \tag{10}$$

where $\omega \equiv \omega_x + i\omega_y$, $\lambda \equiv \Omega_z(C - A)/A$, and Ω is a complex constant of integration. Since the modulus of the right-hand side of Eq. (10) is unity, the projection of $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ on the x, y plane is a constant:

$$|\omega| = |\Omega| = (\omega_x{}^2 + \omega_y{}^2)^{1/2} \equiv \Omega_{xy} \tag{11}$$

From Eqs. (2) and (9)

$$\mathbf{L} = A(\mathbf{i}\omega_x + \mathbf{j}\omega_y) + \mathbf{k}C\Omega_z = A\,\mathbf{n}_{xy}\Omega_{xy} + \mathbf{k}C\Omega_z \tag{12}$$

It is seen that the projection L_{xy} of $\mathbf{L}$ on the x,y plane is